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ABSTRACT

The Mathematics Pentathlon is a tournament of mathematics games held each spring since 1979 to promote the development of mathematical concepts and skills while fostering interaction among the educational community. There are five games in each of four divisions, for grades K-1, 2-3, 4-5, and 6-7. The focus is on active problem solving and critical thinking skills for children at different ability levels. This manual for grades 6-7 first discusses problem solving, educational community involvement, the range of academic performance, and the use of physical models. Then each game is described, with mathematical content, materials, time, goals, starting procedures, game rules, and tournament rules stated, plus a reproduction of the gameboard. The games for this level are Remainder Islands, Prime Gold, Pent 'Em In, Frac Fact, and Fraction Pinball. (MNS)

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A MANUAL OF DIRECTIONS AND OFFICIAL TOURNAMENT RULES

PENT 'EM IN

PRIME GOLD

COACTION PENBALL

REMAINDER ISLANDS

PARC FACT

DIVISION IV
Grades 6-7

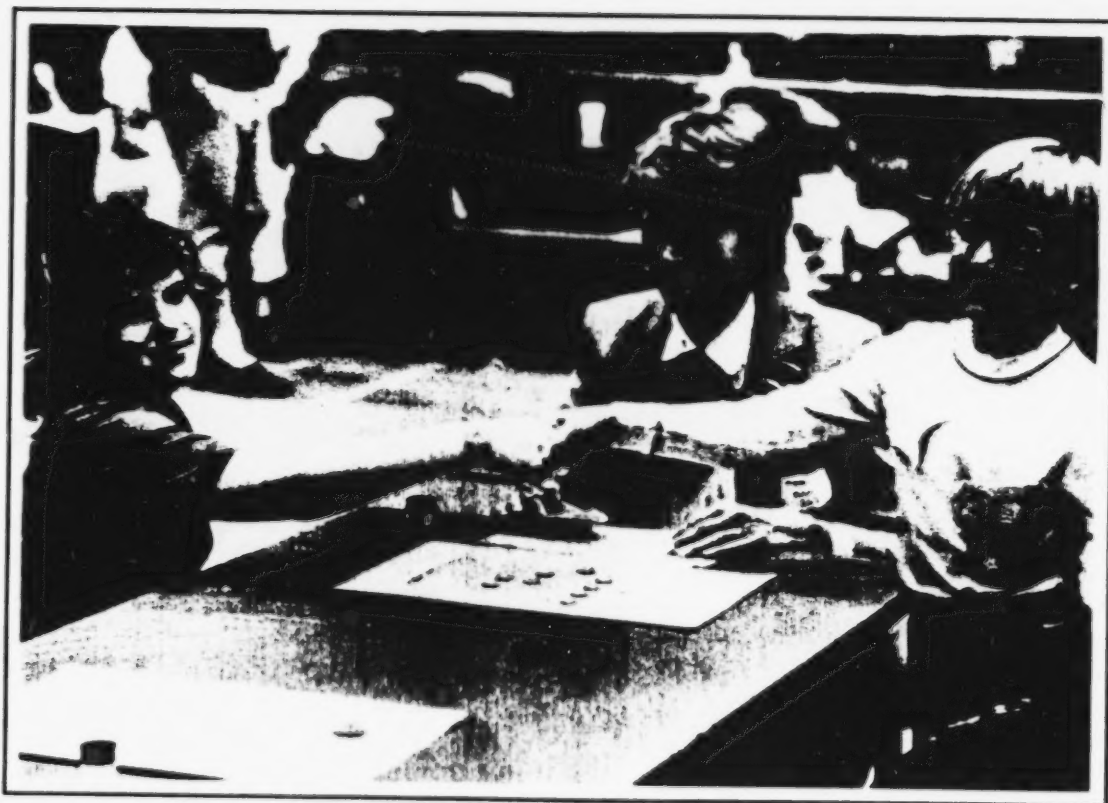
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CENTER FOR DISSEMINATION
OF LEARNING MATERIALS

The Mathematics Pentathlon® provides recognition of those mathletes who have been successful in competing with others. The real winning spirit, however, is best exemplified by the ability of a Mathlete™ to congratulate an opponent on a game well played regardless of the outcome.



MATHEMATICS PENTATHLON®

A MANUAL OF
DIRECTIONS
AND
OFFICIAL TOURNAMENT RULES

DIVISION IV
Grades 6-7

Developed by:
John C. del Regato
Mary E. Gilfeather
Jane A. Schwarm

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DEDICATION



Photo by Jimmy Mack

David



Tom



Kelly



April

The only ladder to the stars is woven with dreams. It is never easy to keep reaching for dreams. Strength and courage can sometimes be lonely friends — but those who reach walk in stardust.

ACKNOWLEDGMENTS

The authors would like to express their indebtedness to the thousands of students, parents, teachers, and administrators who since 1979 have shared their dream and helped them make it a reality. Each new center for the Mathematics Pentathlon® has attracted hundreds of dedicated educators as volunteers for this exciting Mathlete™ tournament.

For their dedication and for their leadership in directing past Mathematics Pentathlon centers the authors are indebted to: Emma Jean Couture, Jane Morton, Donna Nunn, John Peterson, and Stephanie Warren. The Pentathlon could not have succeeded these past years without their guidance and most generous assistance.

It is with great pleasure that we thank the many past directors of Division IV for their warmhearted cooperation and assistance in refining the game rules for tournament competition. They also gave generously of their time to help orient thousands of volunteers to their responsibilities as tournament officials. We gratefully acknowledge Sherry Bash, Pat Blakeley, Carey Chappell, and Paula Weinmann.

For their generous financial contributions to the development of the Pentathlon, we express our thanks to Lilly Endowment and Wickes Foundation. Without their assistance we would not have been able to develop the tournament as rapidly as we have.

From the thousands of volunteers who either gave of their time or personally supported the authors in making the tournament grow, we gratefully acknowledge the contributions of: Bea D'Ambrosia, Jeanne Angst, Darlene Adkins, Pam and Roger Barry, Ellen and Mark Bell, Dorothy Carver, Mary Chappell, Tom Clark, Jim Crane, Richard Crowe, Diane del Regato, Juan and Inez del Regato, Lennie Dees, Sharon Emerson, Betty and James Finkbeiner, Kathleen Fleming, Nancy Goud, Linda Hawa, Christine Hayes, Sue Keene, Mark Keisling, Elaine Kemp, Margaret Knowlton, Paul Lindquist, Mike Metzger, Paula and Jim Minton, Vickie Moody, Robert O'Ana, Susan O'Brian, Jan Palkowski, Hal Peterson, Martha Render, Mike Ropp, Heinz Schwarm, Wayne Taylor, Sara Tomkinson, Ron Weirich, Ben Wilson, Mary Jo Wright, and Barbara Young.

Lastly, we congratulate the Mathletes who enthusiastically responded to the tournament and were placed in the Hall of Fame for their outstanding achievement.

March 15, 1986

John C. del Regato
Mary E. Gilfeather
Jane A. Schwarm

I wish to express my sincere appreciation to John del Regato and Mary Gilfeather for their excellent cooperation in preparing this manual. My sincere thanks also go to Marcia Kazmierzak, Sandra Barger, and Debbie Webb of the PDK staff for the preparation of the materials. I owe a special debt of gratitude to Willard Duckett of the CEDR Center whose constructive suggestions have made my task that much easier. Finally, special thanks to Wilmer Bugher, my predecessor, for not only identifying this program but for his excellent assistance, guidance, and support at all times.

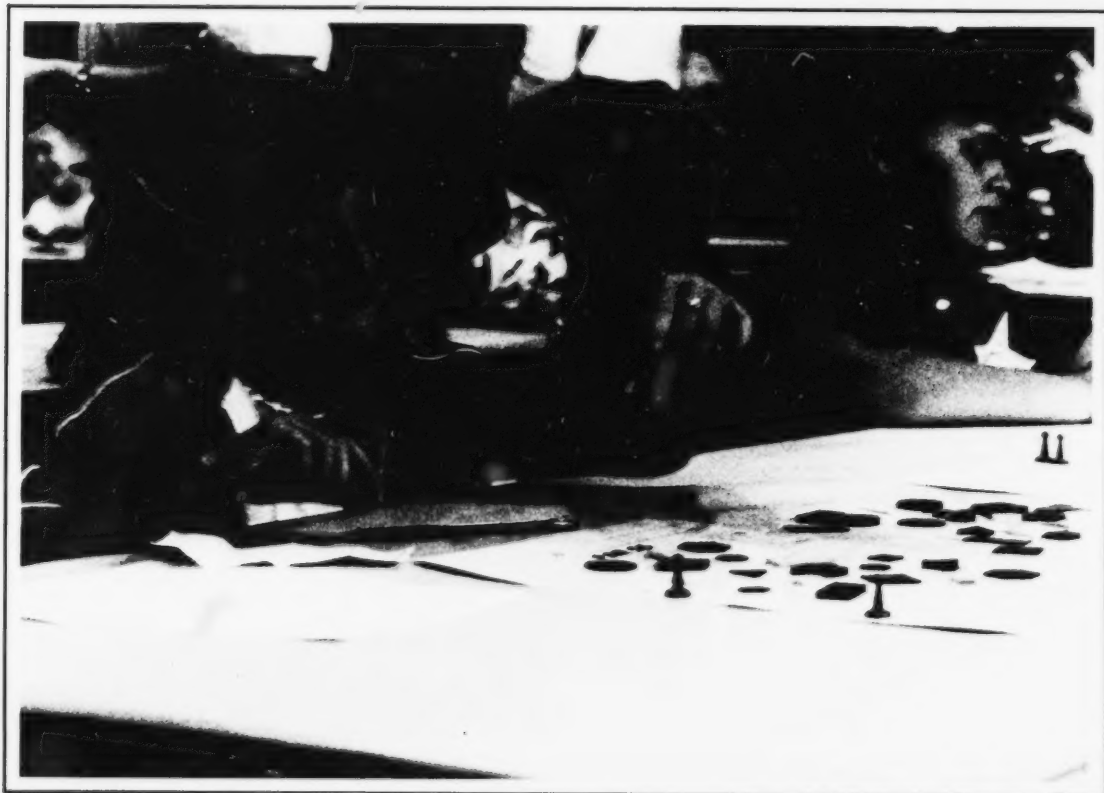
This manual is part of the eighteenth program offered by Phi Delta Kappa's Center for Dissemination of Innovative Programs since the Center was organized in 1972.

March 15, 1986

Neville L. Robertson, Director
Dissemination Center
Phi Delta Kappa

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Photos by Eric Greulich

INTRODUCTION

The Mathematics Pentathlon®: Problem Solving in Action

The Mathematics Pentathlon brings parents, teachers, administrators, and learners together in an educational and social event which promotes the development of mathematical concepts and skills. This tournament of mathematics games held in spring appeals to a wide range of students at different ability levels and fosters interaction among all segments of the educational community. This event is referred to as a Pentathlon because there are 5 games in each of 4 divisions. Each division consists of two grade levels: Division I - grades K-1; Division II - grades 2-3; Division III - grades 4-5; and Division IV - grades 6-7. This event focuses on active problem solving and critical thinking skills. National assessments and international studies have shown that most children lack a true understanding of mathematics on a higher level which involves the development of critical thinking skills, logical and hypothetical reasoning, discerning patterns, and seeing new relationships. With this concern in mind, professional education organizations have often emphasized problem solving and reasoning skills. Continually mounting evidence suggests that the Pentathlon offers an excellent solution to this need.

The Pentathlon was organized as a tournament of mathematics games in 1979 as a tribute to the International Year of the Child. Since then it has become an annual event in the Tri-County area (Saginaw-Bay-Midland) of Michigan. This Center for the Mathematics Pentathlon is now under the leadership of parents and educators who believe it to be a most rewarding educational experience. In 1981 a new Tri-State Center (Indiana, Kentucky, Illinois) was established in Evansville, Indiana. Over the past years both of these centers have provided parents and educators with orientation sessions and in-service education programs related to the tournament. An ever-increasing volunteer support group of teachers, parents, administrators, and students has provided the best evaluative index of the impact this tournament has made.

In 1984 the Indianapolis center for the Mathematics Pentathlon was established. At the same time the National Headquarters for the Pentathlon was organized. Also located in Indianapolis, this headquarters is responsible for assisting other centers and establishing new ones on a national basis. Since then a number of other centers for the Mathematics Pentathlon tournament have developed. It is estimated at this writing that well over 100,000 Mathletes™ nationally are involved in practicing these problem-solving games for tournament competition.

What has caused this dramatic involvement in a problem-solving tournament? Why is the Mathematics Pentathlon growing exponentially in the numbers of individuals involved? The answers to these questions appear to lie in the unique way this tournament has approached the crisis currently faced in mathematics education. Some of the unique features of the Mathematics Pentathlon are: 1) its focus on active problem solving; 2) its involvement of the total educational community; 3) its acceptance of a broad range of academic performance levels by participants; 4) its use of concrete and pictorial manipulatives; and 5) its balanced use of non-verbal and spatial reasoning.

Active Problem Solving

Alvin Toffler and other futurists have brought to our attention that our future is going to involve change at a rate even greater than present generations have experienced.

This implies that future problems will involve a dynamic component which will be undergoing change even as those problems are being studied. It is clear then that a fundamental skill for dealing with the future is active problem solving, i.e., the ability to solve problems which are undergoing change during the process of attempts to resolve them.



The Mathematics Pentathlon® provides experiences in thought processes necessary for active problem solving. Through judicious use of logical analysis, spatial reasoning, computational reasoning, strategic thinking, and chance, the Pentathlon Mathlete™ attempts to resolve problems created by five entirely different two-person game events. Matched with a different opponent in each of the five tournament events, a Mathlete attempts to select a best

option from a cluster of continually changing options. The Pentathlon game events rely to a great extent on the use of strategies. This provides experience in inductive and deductive reasoning through the use of systematic implementation of sequential thought. Such thought patterns are directly related to real life problem-solving situations. For example, the idea of networking is a combination of several different simultaneous approaches to the same problem.

Unlike other contests and competitions which provide individuals with static routine problems for solution by applying known algorithms, the Pentathlon focuses on non-routine types of problem-solving situations that foster the development of active problem-solving skills. But this emphasis on a dynamic approach to solving mathematical problems is not the only characteristic which makes the Pentathlon a unique experience.

Total Educational Community Involvement

The Pentathlon games were designed so that adults as well as children would enjoy them. Just as athletic programs are often supported by the home and school, the Pentathlon nurtures home and school involvement through supporting the efforts of each Mathlete who comes to a tournament. The great interest by both adult and student may be traced in part to the dynamic problem-solving features mentioned above. Parents, teachers, and administrators have found these games to be of educational value because they cover a variety of mathematical concepts and skills such as: basic arithmetic operations, fractions, decimal arithmetic and fraction/decimal conversion, classification and logic, and transformational geometry to name a few.

Currently there are four divisions within this tournament with each division encompassing two grade levels: Division I - grades K-1, Division II - grades 2-3, Division III - grades 4-5, and Division IV - grades 6-7. A new Division V for grades 8 and 9 is presently being constructed. Within each division there are five different games which reflect the mathematical concepts and skills appropriate for that grade level.

During the months of August through December, parents and educators are oriented to the games that comprise the tournament. These individuals then teach their children the games at home and/or in school and prepare and select those Mathletes™ who will register for tournament competition. These same individuals also decide whether or not they will volunteer their services as one of the Pentathlon tournament officials or as one of the many volunteer assistants. Another unique feature of the Mathematics Pentathlon® is the ratio of Mathletes™ to tournament volunteers — 2:1! This type of volunteer support is only possible because of the pride and ownership that parents and educators feel toward this event.

Representatives From a Broad Range of Academic Performance

Currently the Pentathlon consists of a total of 20 different games. Each of these games is either an original creation or some significant modification of a known game. Considerable care has been taken in designing the five different two-person games for each division. Factors which entered into the development process included: 1) creating games sensitive to the developmental differences of students; 2) providing well defined rules and instructions for tournament play; 3) designing games which support and supplement formal mathematics curricula; 4) making the games appealing to both students and adults by blending both strategy and chance within the games; and 5) emphasizing the variety of ways manipulation of physical models can be used in problem solving. The first criterion in this list has enabled the Pentathlon to attract students from a broad range of academic performance. Since the games employ non-verbal as well as the more traditional symbolic and computational aspects, many students who are strong in non-verbal or spatial reasoning abilities are attracted to the Pentathlon games which require this type of thinking. Often a child may be very skilled in this type of reasoning and



yet be either disinterested and/or poorly skilled in the verbal and quantitative skills so valued within formal instructional settings. Many such students have found the Pentathlon experience to be rewarding not only because it values spatial reasoning and non-verbal ability, but because it provides a vehicle for motivating the development of logical and quantitative thinking. The child of average academic performance may also be similarly motivated. And, the student who excels academically is also challenged by the games which require multiple classification and/or geometric and spatial reasoning. The active problem-solving feature discussed above fascinates all. But, the creative use of physical models to represent the underlying mathematical concepts is yet another reason why the Pentathlon has such a broad range of appeal.

Physical Models

The tournament games employ a variety of educational manipulatives. Examples of such materials are pattern and attribute blocks, Cuisenaire rods, pentominoes, dominoes, and fraction bars. Although research studies have indicated the great value these and other physical models have for developing children's mathematical thinking, children often have little or no experience with these valuable learning tools.

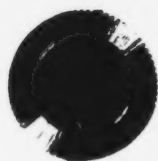
Problem solving most typically requires the solver to build a model or create a picture of the problem. Many students have not been provided with experiences which make use of physical or pictorial models in the development of mathematical ideas. Typically these students lack the necessary imaging for resolving mathematical problems that have a spatial reasoning component. Since all of the mathematical ideas studied in grades K through 12 can be communicated using visual models, it appears most reasonable to enrich a child's experience by the appropriate use of concrete and pictorial instructional manipulatives.

Parents and educators are often surprised by the way in which physical models enable a child to investigate mathematical relationships that were not easily seen with the use of symbols alone. Parents appreciate the value of these objects as they see their child conquer the strategies used in a Pentathlon game. These parents then become most supportive of the need and use of these materials in their child's ongoing formal instruction. Teachers and administrators also gain a renewed respect for the way in which physical and pictorial models enrich the imaging related to mathematics concept development.

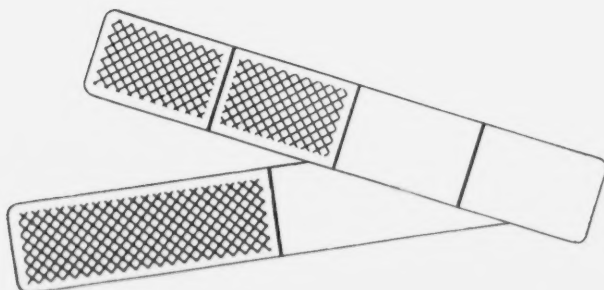
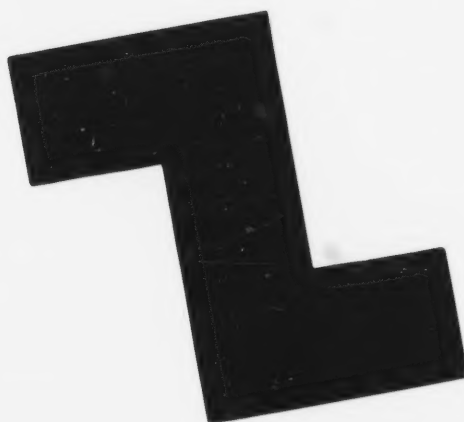
The Future

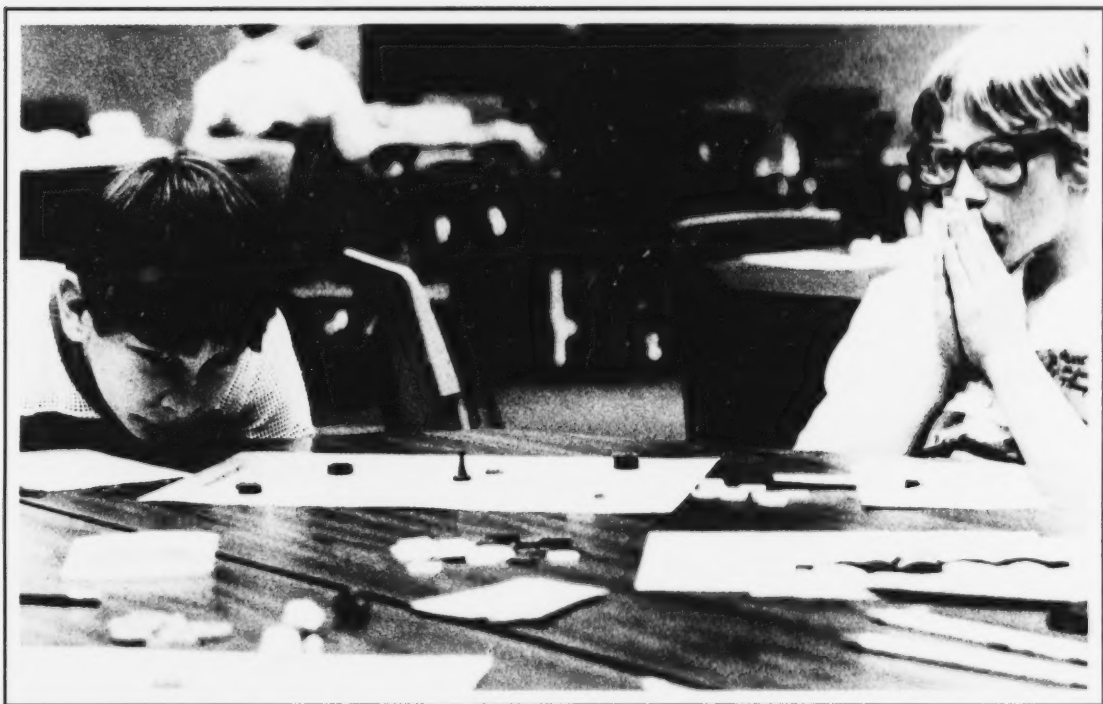
It is through the efforts of many dedicated parents and teachers that the Mathematics Pentathlon® has become an established annual event. All indicators point to a bright future for this unique tournament. If you would like to see a Pentathlon, attend one of the many planned orientation sessions, or arrange your own inservice workshop, call (317) 283-6225 for further information. There is no doubt that the Pentathlon's focus on active problem solving is providing an avenue for children to experience tomorrow's mathematical skills. We can no longer look to the future through a rearview mirror. Rather than a nostalgic return to the past emphasis on the basics of arithmetic memorization and drill, the Pentathlon stresses fundamentals that are future oriented. By acquiring active problem-solving skills, Mathletes™ can look to future challenges with confidence.





GAMES





REMAINDER ISLANDS™

Created by
Dr. John C. del Regato

The operation of division is exercised in this Mathematics Pentathlon® event as each Mathlete™ competes in a race to visit all six hexagonal-shaped islands and, in the process, to acquire the greatest number of chips. Although chance plays some part in a successful completion of this event, each player can exercise certain options which may strategically alter the outcome.

Three specially constructed dice are used in this Mathematics Pentathlon event. Two of these dice are called icosahedra. Each of these dice has twenty sides and displays the values zero (0) to nine (9) twice. The third die is called a dodecahedron. It is a twelve-sided die containing the values one (1) through twelve (12). The icosahedron and dodecahedron are two of the five regular polyhedra known as *Platonic solids* because they were described in detail by Plato around 400 B.C. The ancient Greeks gave mystical significance to the Platonic solids. The icosahedron, with its maximum volume per surface area, represented water — one of the world's four basic elements as conceived by ancient Greeks. The dodecahedron represented the entire universe to the ancient Greeks because they believed that each of its twelve sides corresponded to the twelve signs of the Zodiac.

UNDERLYING

MATHEMATICS/SCIENCE: Division, identifying divisor, dividend, quotient, and remainder, probability, relationship of division to fractions; observational skills, experimentation, inductive and deductive reasoning skills, active problem solving

MATERIALS:

Official Mathematics Pentathlon gameboard, one pawn and forty (40) chips of one color and another pawn and forty (40) chips of a different color, one dodecahedron, two icosahedra dice, and paper and pencil



TIME:

Total tournament playing time is sixty (60) minutes. Each player has one minute to complete a turn. Prior to the tournament, when first learning the game, more time should be granted.

GOAL:

To win, a player must collectively have the most chips on the islands at the end of the game. The game ends when a player succeeds in placing and retaining at least one chip on each island.

TO START:

- A. The dodecahedron die is tossed by each player to determine which player begins. The player with the lower score begins.
- B. The beginning player selects a pawn and 40 chips of the same color. The remaining pawn and 40 chips belong to the opponent. Both players place their pawns into the long narrow area indicated on the gameboard.

RULES:

1. **Alternating Turns and Computing a Quotient:** Players alternate taking turns. Each player in turn tosses all three dice. The two icosahedra dice can be arranged in either of two ways to form a dividend.

$$\begin{array}{r} \text{QUOTIENT} \\ \text{DIVISOR } \overline{) \text{ DIVIDEND}} \end{array}$$

For example, if one of the icosahedra die reads two (2) and the other five (5) the two values could be arranged to read 25 or 52 as the dividend. The dodecahedron die determines the divisor to be used. The player computes a quotient and remainder. The quotient indicates how many triangular-shaped spaces the player's pawn can be moved and the remainder indicates how many chips can be placed onto a visited island.

2. **Pawn Movement, Chip Placement, and Announcing the Division Problem:** When a player obtains a quotient and remainder, each having a value of one or greater, a path to an island must be sought out with exactly as many triangular steps as indicated by the quotient. To move along a path from one triangular step to another, the triangular steps must share a common side. Movement from one triangle to another triangle which shares only one common point is prohibited. The last triangular step in the path must share one of its sides with the hexagonal island (a point or vertex will not do). If such a path is found, the player's pawn is moved along the path counting each triangle aloud until the final one adjoining the island is reached. The movement of the pawn may be cyclical and the path may circle around to move over triangular steps previously used. But, moving back to a triangular step from which a pawn had just been moved is called back tracking and is not allowed. After placing the pawn on the last triangular step adjoining the island, the player places as many chips on the "visited" island as indicated by the remainder of the division problem. The player must announce the divisor, dividend, quotient and remainder of the division problem used.
3. **Announcing Pass:** If a player is unable to find a path to an island when a quotient and remainder with values

greater than one are obtained, "pass" must be clearly announced. The opposing player may begin a turn when pass is announced.

4. **Division Dividend, Pawn Movement of Chip Erosion:**

Should a player toss the dice so that the quotient is one or greater and the remainder is zero, one of the following options may be selected:

- a) The player's pawn is moved the number of spaces indicated by the quotient. The pawn does not have to land on a triangular step adjoining an island in this case.

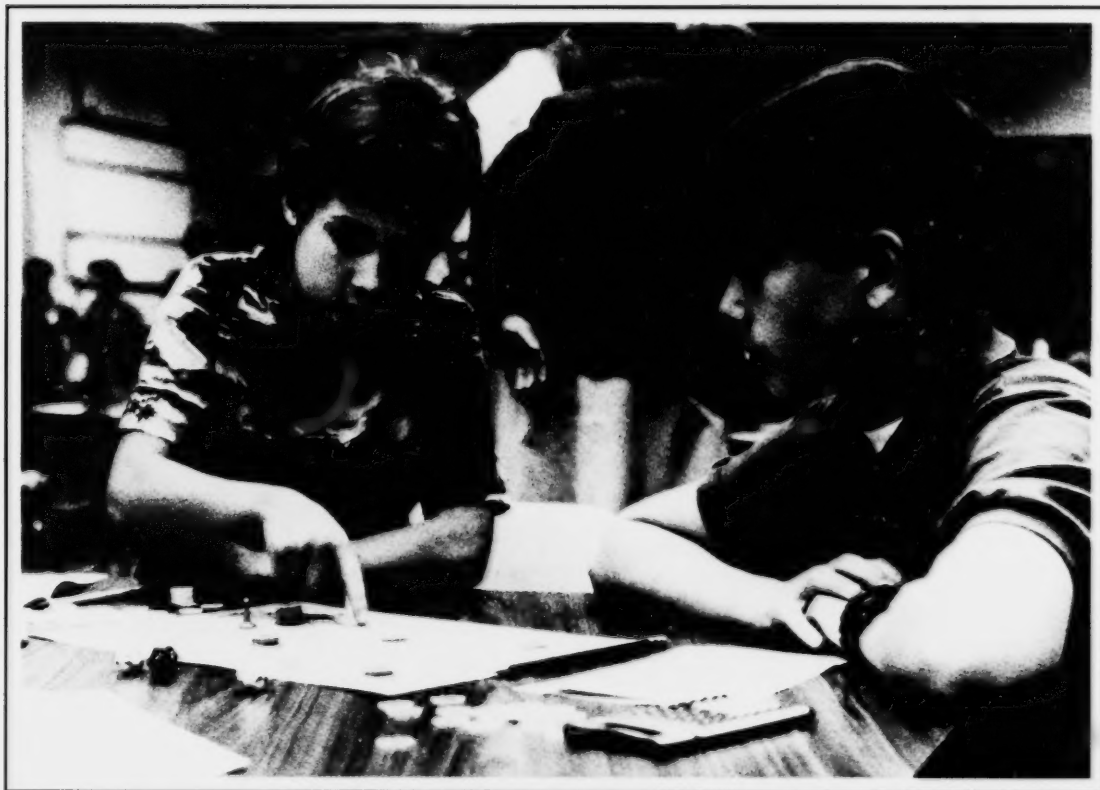


- b) The opponent's pawn is moved by the player the number of spaces indicated by the quotient. In this case also, the pawn does not have to land on a triangular step adjoining an island.
- c) The opponent's chips are removed from the islands under the following conditions:
 - i. If the quotient is less than 20 (i.e. $1 \leq x < 20$), one of the opponent's chips is removed from the gameboard.

- ii. If the quotient is less than 30 but greater than or equal to 20 (i.e. $20 \leq x < 30$), exactly two of the opponent's chips are removed from one of the islands. Both chips removed must be from the same island.
- iii. If the quotient is 30 or greater ($30 \leq x$), exactly three of the opponent's chips are removed from one of the islands. All three chips removed must be from the same island.

To exercise the above options, the opposing player must have at least the number of chips specified. For example, iii can not be implemented if the opposing player has only two chips on one island and one on another. In such an instance, a player may elect option a or b, construct a different division problem, or pass.

- 5. **Discard Pile:** Chips removed from islands in exercising rule 4c are set aside in a discard pile and are not used again during the game.
- 6. **Revisiting an Island:** A visited island containing at least one of a player's chips can not be revisited by that player. If the chips deposited onto an island are removed by

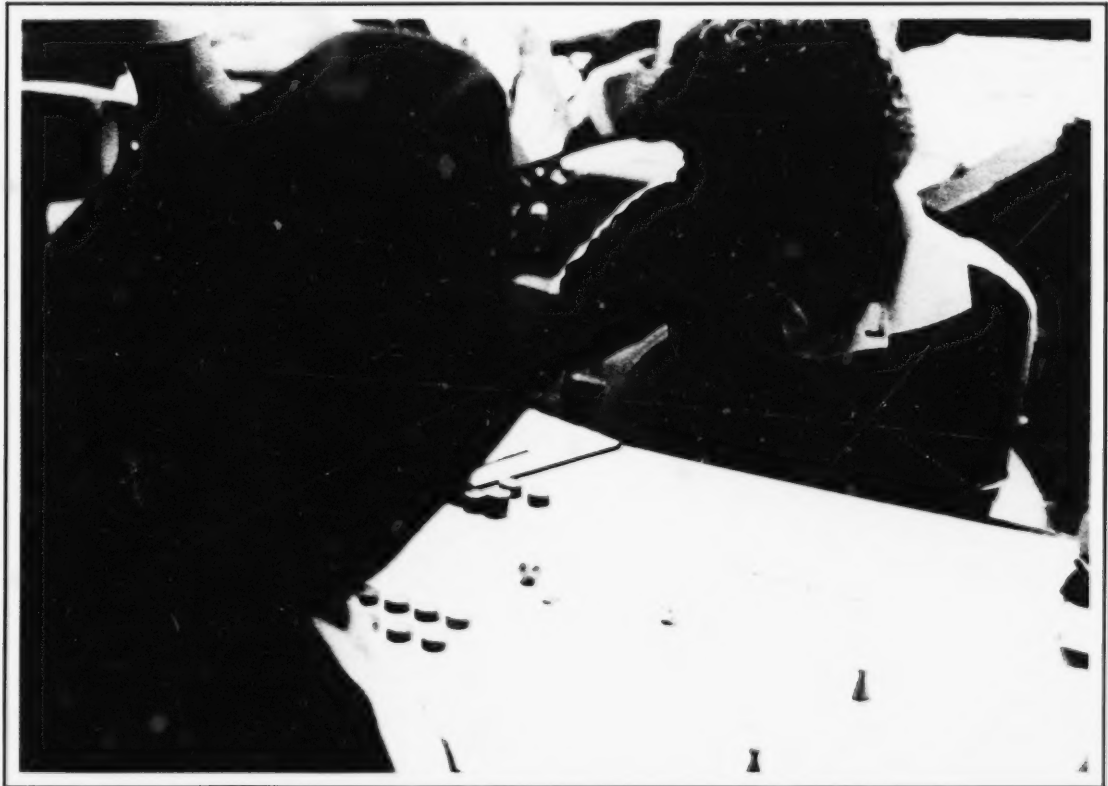




an opposing player (see rule 4c) so that none is left on the island, a player may return to visit the island and deposit more chips onto the island.

7. **Not Executing A Legitimate Move:** A player can elect at any time not to execute a legitimate move dictated by the toss of the dice. The player must clearly announce "pass."
8. **Forty Chip Deposit:** Each player has only 40 chips as a maximum to place onto the islands. If a player uses all 40 chips without concluding the game, rule 4 can be exercised when possible. In all other instances "pass" must be announced.
9. **A No Movement Deposit:** If a player's pawn is on a triangle adjacent to an island that does not contain any of the player's chips and if the player constructs a division problem with a quotient of zero and with a non-zero remainder, the player may deposit the number of chips indicated by the remainder without moving the pawn.
10. **Single Occupancy:** Only one pawn can occupy a triangular step at any time. Also, a player may not use an occupied triangular step as part of a path to an island.

11. **A Round:** A round consists of the beginning player and the second player completing a turn.
12. **Beginning a Turn:** A player's turn begins when the dice are picked up. A player must begin a turn no more than 30 seconds after the opponent has ended a turn.
13. **End of Turn:** A player's turn ends when:
 - a) chips being deposited onto an island are released onto the gameboard; or
 - b) in exercising rule 4a or 4b a player's hand is removed from the moved pawn; or
 - c) a player releases the opposing player's chips into a discard pile in exercising rule 4c; or
 - d) a player announces "pass."



14. **End of Game:** If at the end of a round, one player has succeeded in placing one or more chips on each of the six islands, the game is concluded.
15. **Win:** A player wins when the game has ended and the player has more chips deposited on the islands than the opposing player. Note, that under this ruling it is possi-

ble for the player who concludes the game not to win. The winning player needs only to have the most chips deposited over some of the islands, not necessarily over all six islands. This ruling implies that either player should take care in concluding the game with sufficient chips.

16. **Tie:** A tie is declared if the game is ended and both players have the same number of chips or if both players use all 40 chips without ending the game.

TOURNAMENT RULES:

17. **Challenges:** If a player has violated any of the preceding rules, the opposing player may issue a challenge after a player has ended a turn and before picking up the dice to begin a turn. To indicate that a challenge is being made, a player must announce "challenge". A Mathematics Pentathlon® official must rule on this challenge. In order to do so, an official time out is granted and the official will ask the challenger to state the nature of the challenge. The official must rule on whether the challenge, as stated by the challenger, is correct or incorrect. If the challenge is found to be correct, the gameboard is restored to where chips and pawns were positioned at the beginning of the challenged player's turn and three or less of the challenged player's chips are removed from any island(s) by the challenger and placed into the discard pile. The challenger then begins a turn. If the challenge is found to be incorrect, the challenged player removes three or less of the challenger's chips from any island(s) and the challenger forfeits a turn. A player has only thirty (30) seconds from the end of an opponent's turn in which to announce a challenge. Three consecutive incorrect challenges by a player results in forfeiture of the game.
18. **One Minute and Thirty Second Time Limits:** Each player must complete a turn within one minute. If a player fails to complete a turn within one minute, a Mathematics Pentathlon official will restore the gameboard to where all chips and pawns were positioned at the beginning of that turn and the player will forfeit that turn. If a player does not pick up the dice within their (30) seconds nor challenges the opposing player within this time limitation, the player will forfeit a turn. The third violation of time limit rules results in forfeiture of the game. Mathletes™ may not challenge each other over this rule at the Mathematics Pentathlon. An official at the tournament will decide whether or not this rule is being violated.
19. **Game Time Limit:** At the Mathematics Pentathlon, if the sixty (60) minutes of game time has run out before either

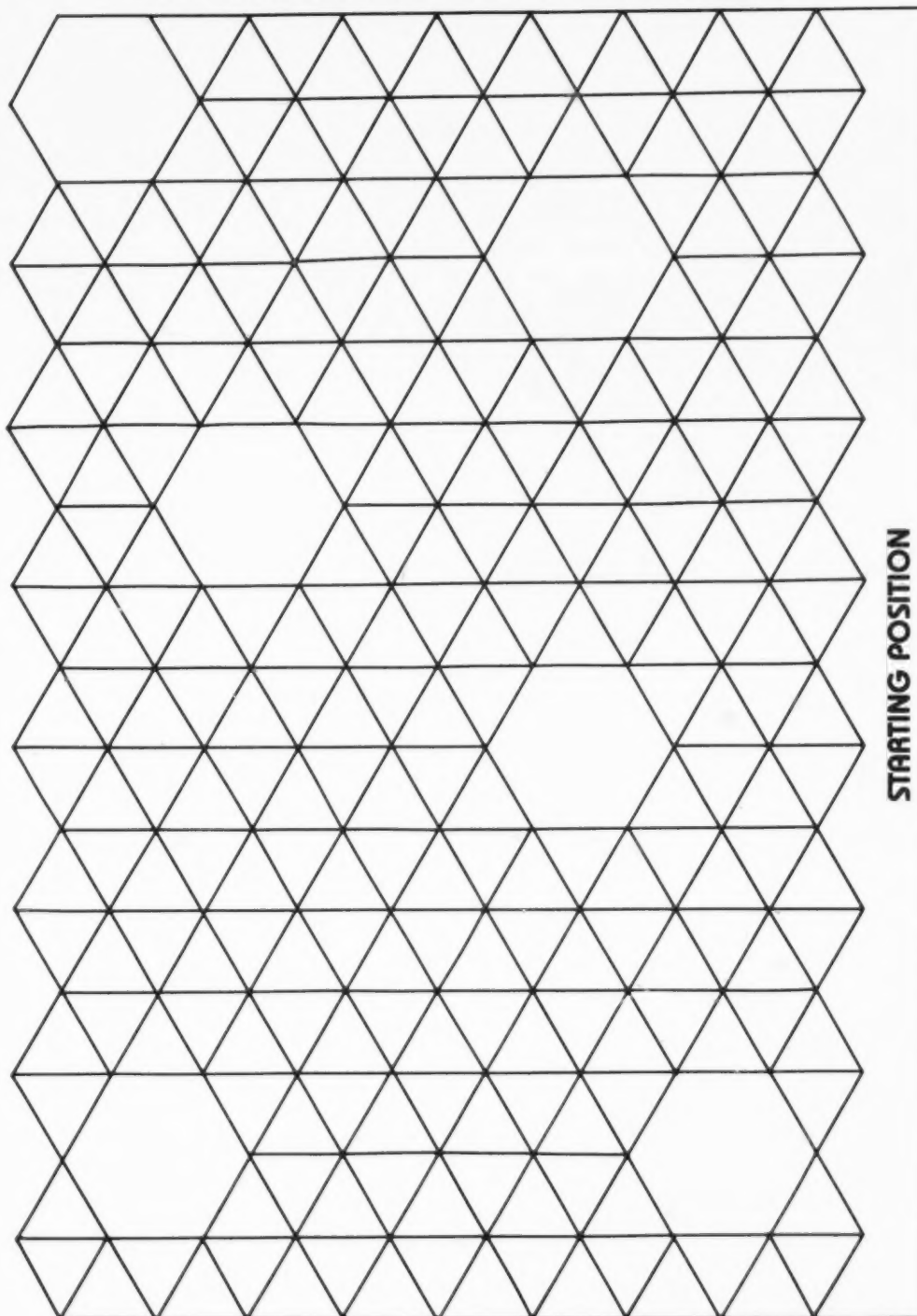
player has ended the game, a round is allowed to be completed. After completing a round, the player with the most chips on the gameboard is declared the winner. If both players have the same number of chips on the gameboard, a tie is declared.

20. **Lists and Calculators:** Lists of division problems may not be used at the tournament. Calculators may be used at the Mathematics Pentathlon® in Division IV, provided each Mathlete™ has one. If one player does not have a calculator, the opposing player may not make use of a calculator.



This page can be used to make an overhead transparency.

REMAINDER ISLANDS

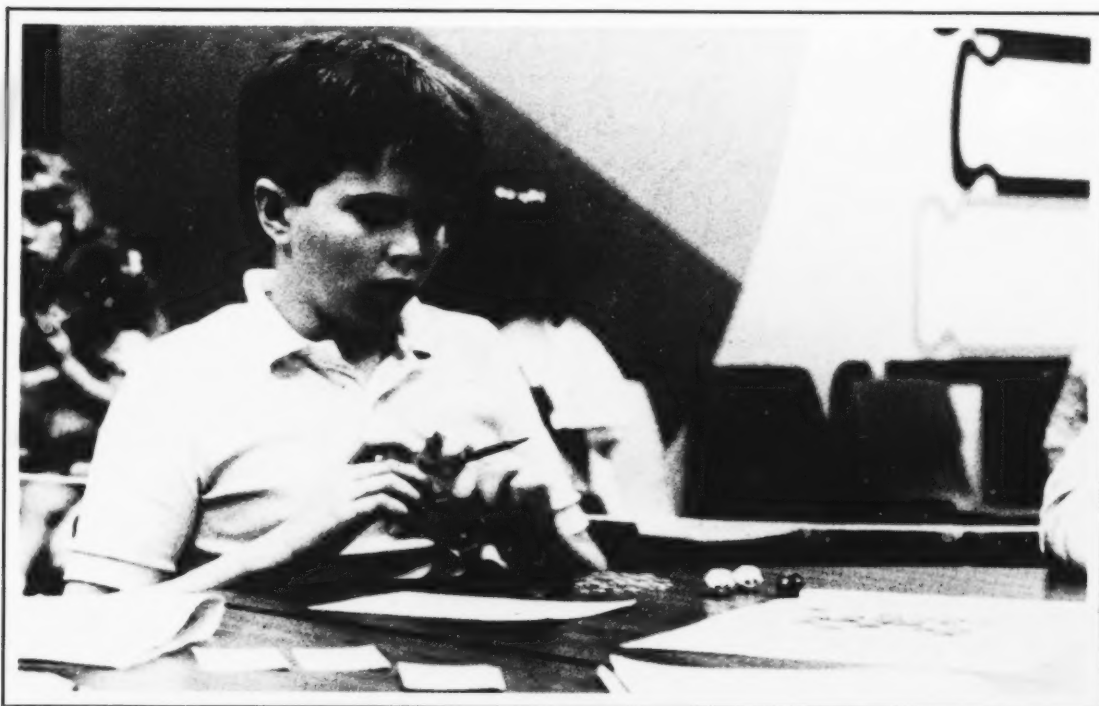


STARTING POSITION

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PRIME GOLD™

Mined by
Dr. John C. del Regato

This strategy-chance game of alignment calls for the ability to plan ahead in constructing a path of chips from one side of the gameboard to the other or from the top to the bottom of the board. A working knowledge of addition, subtraction, multiplication, exponentiation, factorials, and prime and composite numbers is most helpful in succeeding at this Mathematics Pentathlon® event. Before introducing this game to potential Mathletes™, some time should be taken to explain factorials, exponentiation, and prime and composite numbers. Also, a Mathlete would benefit by practicing the basic arithmetic operations with the Pentathlon Division III game CONTIG 60™. KWATRO SINKO™, another Pentathlon game from Division II, can provide useful practice with addition and subtraction.

In 1742, the German mathematician, C. Goldbach, conjectured that every even number except two was the sum of two primes. The Goldbach conjecture becomes most useful to the players of this game as they attempt to use an alternative strategy to win by *mining veins of prime gold*.

A careful examination of the numerals, the prime values and their arrangement on the gameboard leads to the discovery that the probability of arithmetically constructing certain paths is greater than that of constructing others. The probabilities associated with constructing different winning paths varies relative to the positioning on the gameboard. The spiraling of values from the center of the gameboard leads to the effect (described by Stan Ulam) of prime values occurring along an unsymmetrical arrangement of diagonals. A Mathlete's computational reasoning is greatly enhanced through practicing this game.

Three specially constructed dice are used in this Mathematics Pentathlon event. Two of these dice are called icosahedra. Each of these dice has twenty sides and displays the values zero (0) to nine (9). The third die is called a dodecahedron. It is a twelve-sided die containing the values one (1) through twelve (12). The icosahedron and dodecahedron are two of the five regular polyhedra known as *Platonic solids* because they were described in detail by Plato around 400 B.C. The ancient Greeks gave mystical significance to the Platonic solids. The icosahedron, with its maximum volume per surface area, represented water — one of the world's four basic elements as conceived by ancient Greeks. The dodecahedron represented the entire universe to the ancient Greeks because they believed that each of its twelve sides corresponded to the twelve signs of the Zodiac.

UNDERLYING

MATHEMATICS/SCIENCE: Computational reasoning involving addition, subtraction, multiplication, division, greater-than, less-than, equality, factorials, exponentiation, prime and composite numbers, combinatorics, probability, application of Goldbach's conjecture; ob-

servation skills, experimentation, deductive and inductive reasoning, active problem solving

MATERIALS:

Official Mathematics Pentathlon® gameboard, two (2) icosahedra dice and one (1) dodecahedron die, forty (40) chips of one color and forty (40) chips of a different color, pencil and paper



TIME:

Total tournament playing time is one (1) hour. Each player has one minute to complete a turn. Prior to tournament competition, when first learning the game, more time should be granted.

GOAL:

To win, a player must construct one continuous path of chips from top to bottom or from side to side of the gameboard. An alternative way to win is to *mine four veins of prime gold*. A vein of prime gold is defined as three or more chips covering prime numbers in an unbroken straight line.

TO START:

- A. Each player receives 40 chips of one color, a sheet of paper, and a pencil.
- B. Each Mathlete™ tosses one of the icosahedra dice. The player with the lower score begins.

RULES:

1. **Alternating Turns, Placing a Chip, and Announcing Number Sentences:** Players alternate taking turns. Each player in turn tosses all three (3) dice. A number sentence is constructed by using basic arithmetic operations, factorials, and/or exponentiation on the three (3) values obtained from the dice. For example, with 2, 3, and 4 a player might construct any one of the following:

$$\begin{array}{ll}
 2 + 3 + 4 = 9 & (4 + 3)^2 = 49 \\
 2 \times 3 - 4 = 2 & 4! + 2 + 3 = 29 \\
 (3 \times 4) \div 2 = 6 & (2 + 3)! - 4! = 96 \\
 4 + 2^3 = 12 & 2(4!) - 3 = 45
 \end{array}$$

And with 2, 2, 3 a player could construct $(2^2)^3 = 64$ or $(3!)^2 \times 2 = 72$.

Possible constructions are limited to the four basic arithmetic operations, exponentiation (powers), and factorial usage. A result is obtained (such as 2, 6, 9, 12, 29, 45, 49, 64, 72, or 96 in the examples above) and covered on the gameboard by placing a chip on an unoccupied space of identical value. The player must clearly announce the number sentence used before releasing hold (touch) of the chip placed onto the gameboard.

2. **Double Prime Placement:** In the case of an even value greater than 6 (such as 12, 64, 72 and 96 in the above examples) the player may elect to cover two different prime numbers which sum to the designated value. For example, with the value of 12, the player could choose to cover both 5 and 7 instead of just placing one chip on the space numbered 12. In addition to the number sentence used to construct the value, the player must also announce the two prime numbers which sum to the value constructed before releasing hold (touch) of the chips placed onto the prime numbers.



3. **Prime Gold Vein of Length Three:** A prime gold vein of length three has been "mined" when a player succeeds in placing chips on top of three prime numbers connected along a diagonal. One of the opponent's chips covering a composite number may be removed from the game-board and given back to the opponent when a player mines a vein of prime gold exactly three chips in length. On another turn, this vein may be extended by placing additional chips on prime numbers which are connected to and lie along the same diagonal vein. For exam-



ple, the vein composed of 7, 23 and 47 could be extended by placing a chip on 19 or 79 (see rules 4 and 5).

4. **Prime Gold Vein Length Four:** A prime gold vein of length four has been "mined" when a player succeeds in placing chips onto four prime numbers connected along a diagonal. Two of the opponent's chips covering composite numbers may be removed from the gameboard and returned to the opponent when a player mines a vein of prime gold exactly four chips in length. Two of the player's chips may be placed onto the composite numbers which were covered by the removed chips. The opponent must have at least two composite numbers covered with chips in order to exercise this rule.
5. **Prime Gold Vein of Length Five:** A prime gold vein of length five has been "mined" when a player succeeds in placing chips onto five prime numbers connected along a diagonal. As in rule 4, two of the opponent's chips covering composite numbers may be replaced by two of the player's chips, when a player mines a vein of prime gold exactly five chips in length. Also, a player may remove one of the opponent's chips which covers a prime

number. All chips removed are returned to the opposing player. The opponent must have at least two composite numbers covered with chips and at least one prime number covered in order to exercise this rule.

6. **Shafts of Five or More Chips:** A shaft has been constructed when a player succeeds in placing at least five chips in a straight continuous connected *row* or *column*. When a player has succeeded in constructing a shaft of exactly five chips, any two (2) or fewer of the opponent's chips may be removed from the gameboard. A shaft can be extended by placing additional chips into adjoining spaces along the same row or column. Any three (3) or fewer of an opponent's chips may be removed when a player succeeds in constructing a shaft of six or more chips in length. If a player simultaneously constructs two shafts of five or more chips in length, the total number of opponent's chips to be removed is in accord with the above rules. For example, if a row shaft of exactly five chips and a column shaft of seven chips are simultaneously constructed, the player may remove 5 (2 for the row shaft and 3 for the column shaft) or fewer of the opponent's chips. All chips removed are returned to the opponent.



7. **A Round:** A round consists of the beginning player and the second player completing a turn.
8. **Beginning a Turn:** A player's turn begins when the dice are picked up. A player must begin a turn no more than 30 seconds after an opponent has ended a turn.
9. **End of Turn:** A player's turn ends either:
 - a) when a chip being placed onto the gameboard has been released by the player; or
 - b) when chips removed from the gameboard are released to the opponent.
10. **Win/Tie:** At the end of a round, the player who has constructed four veins of prime gold or constructed an unbroken path of chips from one side of the gameboard to the opposite side or from the top to the bottom of the gameboard wins. This win must be announced by the player before the opposing player begins a turn. The chips constructing a winning path must be of the same color and must be in squares sharing a common side or vertex. The path need not be straight. It may weave back and forth. If both players announce winning situations at the end of a round, a tie is declared.





TOURNAMENT RULES

11. **Challenges:** If a player has violated any of the preceding rules, the opposing player may issue a challenge after a player is no longer touching the last chip placed onto the gameboard or in the case of chip removal, the last chip returned to the opponent and thus ending a turn. To indicate that a challenge is being made, a player must announce "challenge." A Mathematics Pentathlon® official must rule on this challenge. In order to do so, an official time out is granted and the official will ask the challenger to state the nature of the challenge. The official must rule on whether the challenge, as stated by the player, is correct or incorrect. If the challenge is found to be correct, the gameboard will be restored to where the chips were positioned at the beginning of the challenged player's turn and the challenger will begin a turn. If the challenge is found to be incorrect, the challenger will forfeit a turn. A player has only thirty (30) seconds from the end of an opponent's turn in which to announce a challenge. Three consecutive incorrect challenges by a player results in forfeiture of the game.
12. **One Minute and Thirty Second Time Limits:** Each player must complete a turn within one minute. If a player fails

to complete a turn within one minute, a Mathematics Pentathlon® official will restore the gameboard to where all chips were positioned at the beginning of that turn and the player will forfeit that turn. If a player does not pick up the dice within thirty seconds nor challenge the opposing player within this time limitation, the player will forfeit a turn. The third violation of time limit rules results in forfeiture of the game. Mathletes™ may not challenge each other over this rule at the Mathematics Pentathlon. An official at the tournament will decide whether or not this rule is being violated.

13. **Game Time Limit:** At the Mathematics Pentathlon if the sixty (60) minutes of game time has run out before either player has ended the game, a tie is declared. If a player has tossed the dice and is in the process of completing a turn when game time is called, a round is allowed to be completed.
14. **Lists and Calculators:** Lists of prime numbers may not be used at the tournament. Calculators may be used at the Mathematics Pentathlon in Division IV, provided each Mathlete has one. If one player does not have a calculator, the opposing player may not use a calculator.

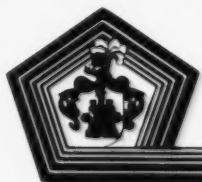


PRIME GOLD

101	100	99	98	97	96	95	94	93	92	91
102	65	64	63	62	61	60	59	58	57	90
103	66	37	36	35	34	33	32	31	56	89
104	67	38	17	16	15	14	13	30	55	88
105	68	39	18	5	4	3	12	29	54	87
106	69	40	19	6	1	2	11	28	53	86
107	70	41	20	7	8	9	10	27	52	85
108	71	42	21	22	23	24	25	26	51	84
109	72	43	44	45	46	47	48	49	50	83
110	73	74	75	76	77	78	79	80	81	82
111	112	113	114	115	116	117	118	119	120	121

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PENT 'EM IN™

Created by
Dr. John C. del Regato

This Mathematics Pentathlon® event is an action packed entrapment game which makes use of pentominoes. Pentominoes consist of all the different shapes made of five square tiles in which at least one edge of each tile is shared in common with some of the other tiles. Many fascinating investigations involving area and perimeter can be generated by the use of pentominoes. In Division III, these shapes along with other polyominoes are used in an event called Juggle™. In this event, the pentominoes are used to actively represent the transformational geometry concepts of rotation, reflection, and translation. These concepts are actively represented by rotating, flipping, and sliding the different pentominoes once they have been positioned onto the gameboard. A Mathlete™ develops an excellent ability to visualize the effects of rotating, flipping, or sliding the geometric shapes used in this game after having investigated these geometrical operations and having repeatedly made use of them. The transformational geometry concept of translation is limited in this game to only vertical, diagonal, and horizontal translations. Thus, a player is allowed to slide a pentomino only vertically or horizontally. The four chips used in this game can be slid diagonally as well as horizontally and vertically.

Each game is unique because of the variety of choices a player has in initially selecting the five pentominoes used for the game and the initial placement of those pentominoes. In first learning this game, a Mathlete should explore the effects of rotating, flipping, and sliding each of the different pentominoes.

UNDERLYING

MATHEMATICS/SCIENCE: Transformational geometry concepts of translation, rotation, and reflection, spatial reasoning, investigation of structural relationships; observation skills, experimentation, deductive and inductive reasoning, active problem solving

MATERIALS: Official Mathematics Pentathlon gameboard, two (2) sets of pentominoes (each set a different color), four (4) chips — two (2) the same color as one of the pentomino sets, the other two (2) chips the same color as the second set of pentominoes

TIME: Total tournament playing time is one (1) hour. Each player has one (1) minute to complete a turn. Prior to tournament competition, when first learning the game, more time should be granted.

GOAL: To win, a player must entrap all of the opponent's pentominoes so that none of the opponent's five (5) selected pentominoes can be moved by a slide, flip or rotation.



TO START:

- A. One player is asked to select a starting color. The second player selects one (1) chip of either color and out of the first player's view places one (1) chip in each hand. The first player then selects one of the second player's hands and takes the chip. If the chip was of the starting color, the player begins the game. If not, the second player begins the game.
- B. All pentominoes and the two (2) chips having the starting color are given to the beginning player. The other set of pentominoes and two (2) chips of similar color are given to the other player.
- C. The beginning player places both chips onto the shaded circles at the center of the gameboard. The opposing player places both chips on the unshaded circles.

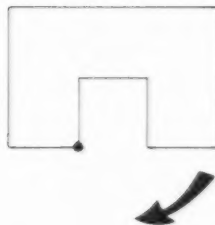
RULES:

1. **Alternating Turns and Pentomino Placement:** Players alternate taking turns by placing one of their pentominoes onto the gameboard. The pentominoes must be placed within the perimeter of the 9" by 13" space gameboard. The pentominoes must totally cover five (5) squares and no pentomino may be placed over the shaded squares

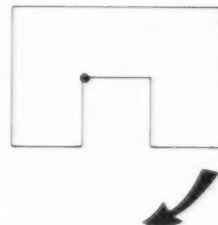
on the gameboard nor may they cover the chips. A player may not place a pentomino onto a square which shares a common side (edge) with a square that is occupied by an opposing player's pentomino. A player's pentomino may touch the corner of an opponent's pentomino but may not be placed edge to edge. A player's pentomino may be placed to share a common side (edge) with another of the player's pentominoes. A player's pentomino may be placed onto any squares surrounding the spaces occupied by the opponent's chips. Players continue to alternate placing pentominoes onto the gameboard until each player has selected and placed five (5) pentominoes onto the gameboard. If a player cannot place a pentomino in this manner, and is challenged, the player forfeits the game.

2. **Pentomino Movement:** After both players have positioned five (5) of their pieces onto the gameboard, they may move a pentomino to another location by using a rotation around an external corner (vertex),

CORRECT

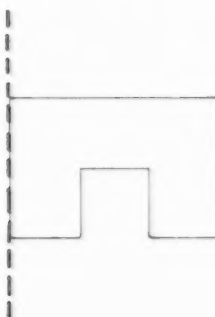


INCORRECT

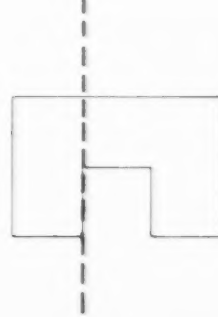


a flip around an external edge,

CORRECT



INCORRECT



or a slide along a vertical or horizontal direction in a straight unobstructed path. A player's pentominoes may now be moved so that they share a common edge with the opponent's pentominoes. Pentomino pieces may be flipped over or rotated over chips, shaded areas, or parts of another pentomino. They may not be slid over spaces occupied by chips or pentominoes or over shaded squares. A pentomino may be slid as many spaces as a player desires. Also, a player may rotate a pentomino off the gameboard as long as it is rotated back onto the gameboard and is positioned within the rectangular region of the playing area.

3. **Chip Movement:** After both players have positioned five (5) of their pieces onto the gameboard, they may move a chip instead of a pentomino during a turn. Chips may be slid in a horizontal, vertical, or diagonal fashion along an unobstructed path as far as is considered desirable by the player.





4. **No Consecutive Chip Movements Allowed:** A player may not make two chip movements in consecutive turns. If a chip has been moved during one turn, the player must move a pentomino piece in the next move.
5. **Starting Circles May Be Covered:** Playing pieces may be moved into or through unoccupied starting circles.
6. **No Double Occupancy:** Two playing pieces may not occupy the same space on the gameboard.
7. **Touch Move:** Once a piece has been touched, it must be moved. If it is determined that there is no legitimate move for that piece, the player must remove one chip from the gameboard and forfeit that turn. The third violation of this rule results in forfeiture of the game. Chips removed from the gameboard are not to be used for the remainder of the game.
8. **Beginning a Turn:** A player's turn begins when the opposing player has released hold of a playing piece that has been either placed or moved on the gameboard.



9. **End of Turn:** A player's turn ends when the player has released hold of a playing piece that has been either placed or moved on the gameboard.
10. **Win:** A player wins when it is the opponent's turn and the opponent's pentominoes are entrapped so that none of the five (5) pentominoes can be moved using a flip, slide, or rotation.

TOURNAMENT RULES:

11. **Challenges:** If a player has violated any of the preceding rules, the opposing player may issue a challenge after a player has ended a turn. To indicate that a challenge is being made, a player must announce "challenge". A Mathematics Pentathlon® official must rule on this challenge. In order to do so, an official time out is granted and the official will ask the challenger to state the nature of the challenge. The official must rule on whether the challenge, as stated by the player, is correct or incorrect. If the challenge is found to be correct and the rule does not specify a consequence, the game-

board will be restored to where the playing pieces were positioned at the beginning of the challenged player's turn and the challenger will begin a turn. If the challenge is found to be incorrect, the challenger will forfeit a turn. Three consecutive incorrect challenges by a player results in forfeiture of the game.

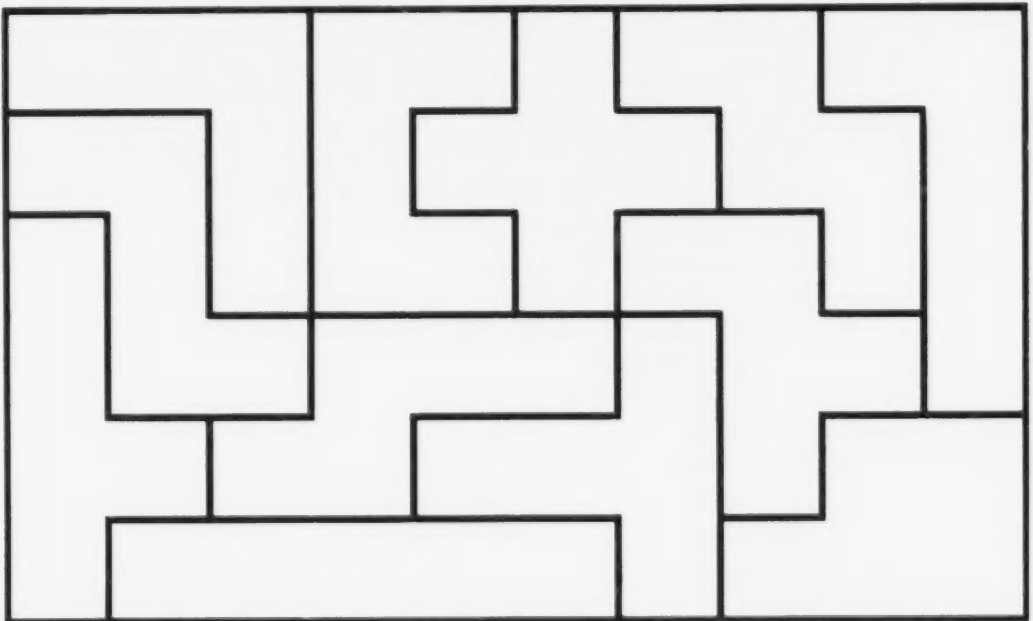
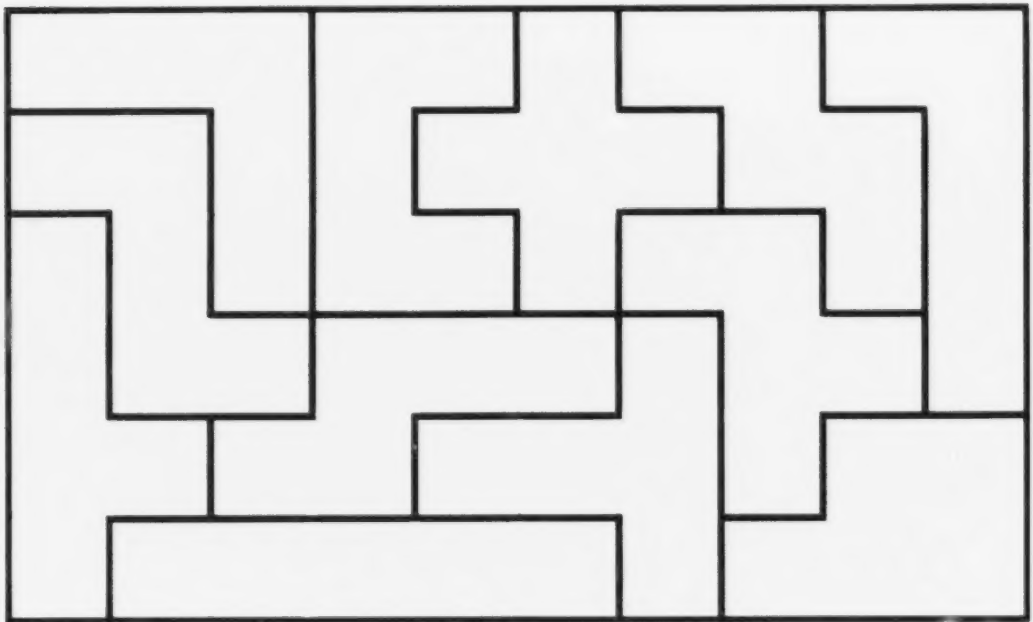
12. **One Minute Time Limits:** Each player must complete a turn within one minute. If a player fails to complete a turn within one minute, a Mathematics Pentathlon® official will restore the gameboard to where all chips were positioned at the beginning of that turn and the player will forfeit that turn. The third violation of time limit rules results in forfeiture of the game.
13. **Game Time Limit:** At the Mathematics Pentathlon, if the sixty (60) minutes of game time have run out before either player has won, a tie is declared. If a player is in the process of completing a turn when game time is called, that turn will be allowed to be completed.



GAMEPIECES FOR PENT 'EM IN

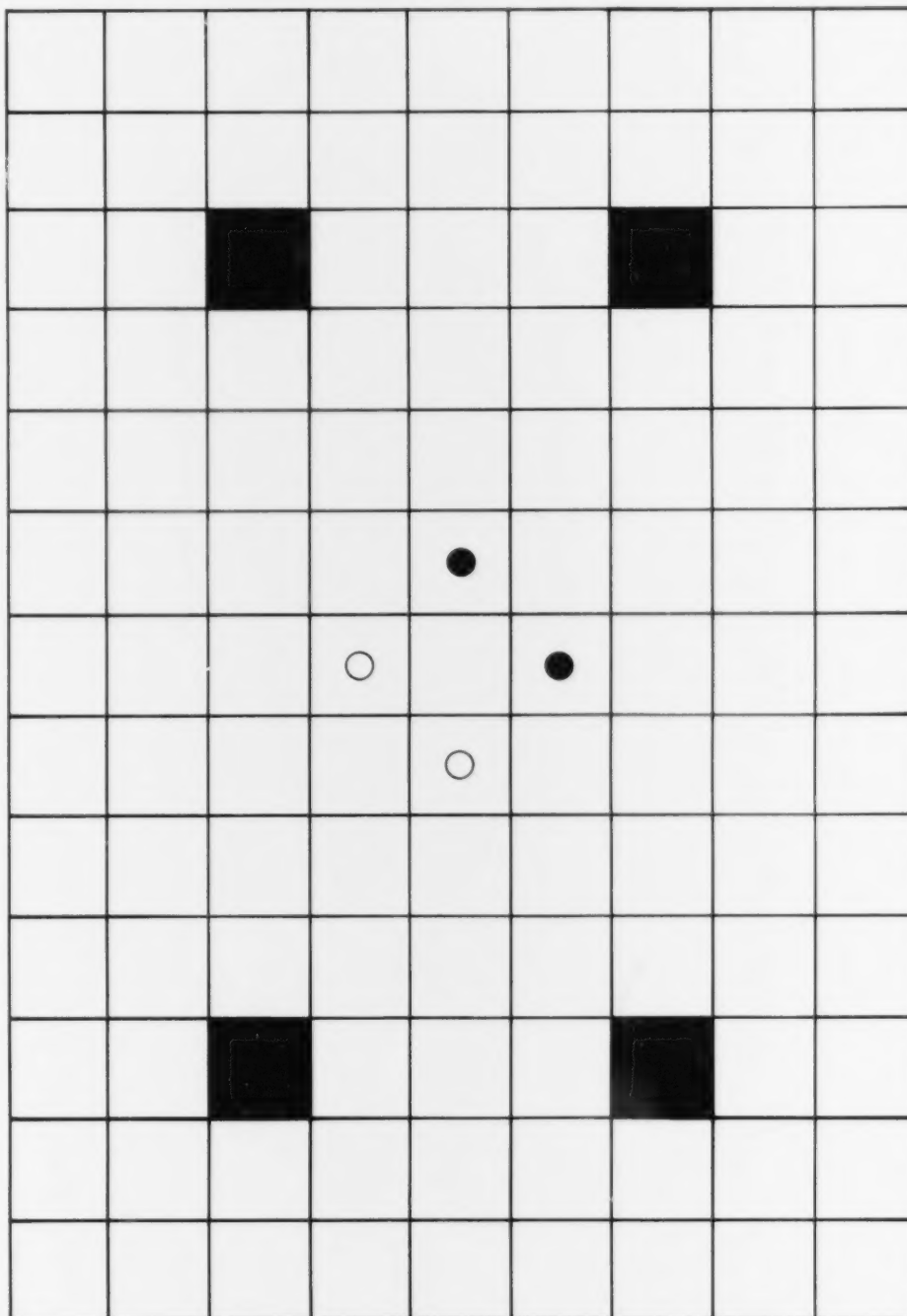
DUPLICATE THIS PAGE

Please color each set of pentominoes a different color.



This page can be used to make an overhead transparency.

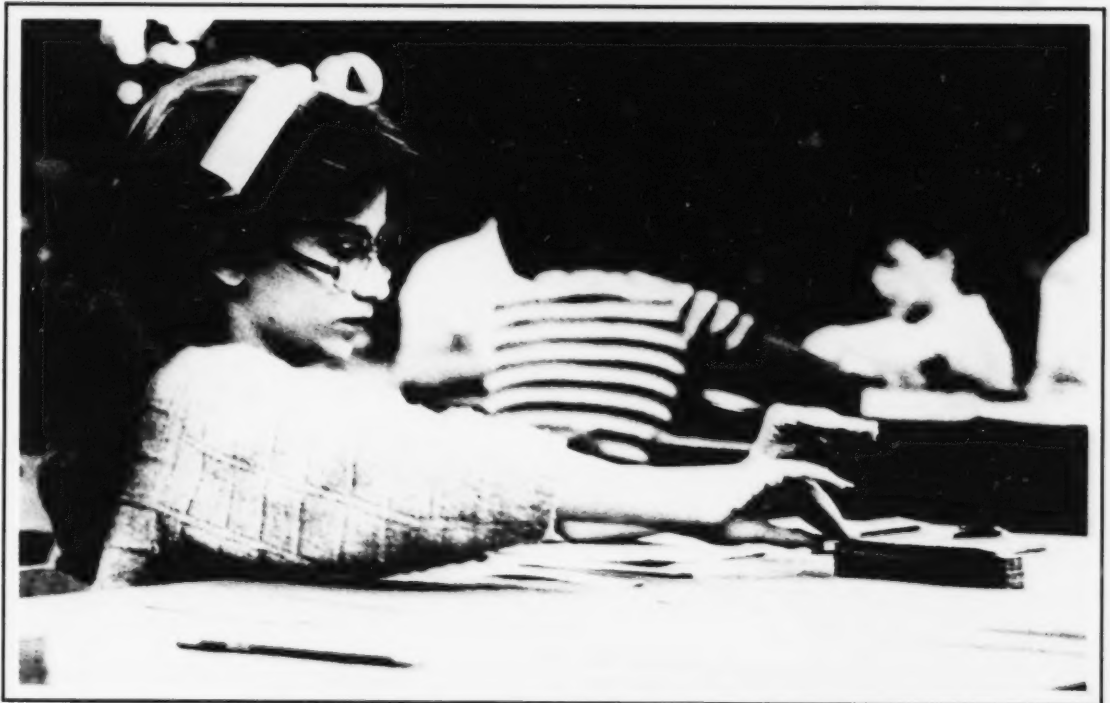
PENT 'EM IN



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FRAC FACT™

Created by
Dr. John C. del Regato

Any of the basic arithmetic operations can be used in this Mathematics Pentathlon® event which challenges each Mathlete™ to discover an appropriate fraction fact. A set of sixty-four (64) fraction bars is used to represent various fractions. Players compete to accumulate as many fraction bars as possible.

UNDERLYING

MATHEMATICS/SCIENCE: Computational reasoning involving fraction addition, subtraction, multiplication, and division, inequality and equality; observation skills, experimentation, deductive and inductive reasoning, active problem solving

MATERIALS:

Official Mathematics Pentathlon gameboard, a set of sixty-four (64) fraction bars (2 each of those pictured in the gamepieces section), paper and pencil

TIME:

Total tournament playing time is one (1) hour. Each player has sixty (60) seconds to complete a turn. Prior to the turn, thirty (30) seconds are allowed to capture an opponent's trick. When first learning the game, more time should be granted.

GOAL:

To win, a Mathlete must have accumulated more fraction bars than the opposing player at the game's conclusion. The game is concluded when there are insufficient fraction bars in the draw pile to replenish a player's hand.

TO START:

- A. The deck of sixty-four (64) fraction bars is thoroughly shuffled and stacked face down in the designated area on the gameboard to form a draw pile.
- B. A player is chosen (at the Mathematics Pentathlon this player is selected by the official) to pick the top fraction bar from the draw pile and to place it face up onto one of the designated starred areas in the center of the gameboard. The opposing player then draws the top fraction bar from the draw pile and places it face up onto the other designated starred area.
- C. The player selecting the fraction bar of least value starts the game by dealing five (5) bars in alternating fashion to both players. Should both players turn over fraction bars of equivalent value, the second player to draw from the pile would deal out the five (5) bars to each player. The five (5) bars dealt to each player are arranged face up in front of each player in the designated unstarred

spaces on the gameboard. After the dealer has distributed five (5) bars to each player, the opposing player begins play.

RULES:

1. **Alternating Turns Forming Tricks and Announcing Arithmetic Sentence:** Players alternate in taking turns. Players, in turn, search among their own five vertically placed fraction bars for two which when combined by using one of the four basic arithmetic operations (addition, subtraction, multiplication, division) yield a result represented by one of the two fraction bars lying horizontally between the two players. When two bars are found which can be combined in such a manner, the player announces the arithmetic sentence to the opposing player, selects the two bars from the player's fraction bars and the bar from the center to which it was matched to form a trick, and places this trick of three bars face down into a scoring pile to the right of the player. The player then draws three bars from the top of the draw-pile, selects two of these bars to replenish the player's five bars and places the third bar into the center starred area on the gameboard.
2. **Replenishing a Hand:** During the first 30 seconds of a player's turn an option may be exercised to discard from one (1) to all five (5) of the player's fraction bars and to replenish these by drawing sufficient bars from the draw-pile to maintain a player's hand at five bars. The number of bars in a player's hand must always be kept at five. The discarded bars are stacked in a discard pile on the player's left hand side. Players have their own discard piles. After exercising the replenishment option, the player must use the remaining time to attempt to form a trick. The replenishment option can be used only once within any given turn.
3. **Touch Pass Signal:** During a turn, should a player be unable to find a suitable combination of fraction bars to form a trick, the opposing player can be signaled that the player wishes to pass by the player touching one of the two fraction bars in the center of the gameboard. Touching a bar in the center of the playing area indicates that a player has passed that turn.
4. **A Forced Discard:** Should a player pass a turn (see rule 3) without having replenished a hand, the opposing player may signal a pass back to the player. This forces the player to replenish at least one of the fraction bars in the player's hand and to place the bar or bars selected into the player's discard pile.

5. **Capturing a Fact:** Should a player pass when a trick could have been made, the opposing player has thirty (30) seconds to capture the trick. To do this the opposing player must announce "FACT". The arithmetic sentence which combines two of the player's bars and results in a fractional value represented by one of the two bars in the center of the gameboard, is captured by the opponent. After placing this captured trick into the opposing player's scoring pile, the opposing player draws three bars, deals two bars to replenish the player's hand and places the third into the central designated area on the gameboard. The opposing player then continues to complete a turn.



6. **A Round:** A round consists of the beginning player and the second player completing a turn.
7. **Beginning A Turn:** A player's turn begins:
- a) when the opposing player has released hold of the fraction bar placed into the center of the gameboard;
 - b) when "FACT" is declared and the player has released hold of the fraction bar placed into the center of the gameboard. In this case, a player then begins a regular turn;

- c) when the opposing player signals a pass by touching one of the two center fraction bars;
 - d) in the case of a correct challenge (see rule 13) when the challenger receives and places the opponent's trick into the challenger's scoring pile. The challenger begins a regular turn;
 - e) in the case of an incorrect challenge (see rule 13) when the challenger releases hold of a bar placed into the challenger's discard pile. The challenged player begins a turn.
8. **End of Turn:** A player's turn ends:
- a) when the player has released hold of the fraction bar placed into the center of the gameboard. In the case that "FACT" is declared, a player's turn ends upon completion of the regular turn.
 - b) when the player signals a pass by touching one of the two center fraction bars.
9. **End of Game:** If there is an insufficient number of bars in the draw pile to replenish a player's hand and the fraction bar selected from the center of the gameboard, the game is concluded. The player may complete a turn but after the trick of three bars is added to the scoring pile the game is officially over.





10. **Win/Tie:** At the end of the game, players count up the number of bars in their scoring piles. After subtracting the number of bars in each of their discard piles from those in the scoring piles, the winner is the player with the larger difference. If both players have the same difference after completing the above comparison subtraction, a tie is declared.
11. **Winning the FRAC FACT™ Event:** At the Mathematics Pentathlon® the Mathlete™ who wins two (2) out of three (3) games or who wins one (1) game and ties in two (2) games is the winner of the FRAC FACT event.
12. **A Tie in the FRAC FACT Event:** If a Mathlete at the Pentathlon ties all three games in a series, a tie will be declared. If one Mathlete wins one game and the opposing Mathlete wins another game in the series and a third game in the series is tied, a tie will be declared for the FRAC FACT event.
13. **Challenges:** If a player violates any of the preceding rules, the opposing player may issue a challenge. To indicate a challenge is being made, a player must announce "challenge". A Mathematics Pentathlon official must rule on this challenge. In order to do so, an official time out will be granted and the official will ask the challenger to

TOURNAMENT RULES:



state the nature of the challenge. The official must rule on whether the challenge, as stated by the player, is correct or incorrect. A player may be challenged for stating an arithmetic sentence in forming a trick or for stating an incorrect arithmetic sentence. (WARNING: Equivalent expressions for the fraction representations can be used in stating the arithmetic sentence.) The challenger does not have to state the correct equation. A challenger has thirty (30) seconds from the end of an opposing player's turn to declare "challenge" and must do so before forming a trick. A challenge to a captured FRAC FACT trick (see rule 5), as well as any other trick, must be made during this time interval.

If a challenge is found to be correct, the three bars forming the trick or tricks are awarded to the challenger and are placed onto the challenger's scoring pile. The challenger must first replace the three trick cards on the gameboard and then begins a turn. If the challenger is found to be incorrect, the challenger forfeits that turn. Three consecutive incorrect challenges by the player results in forfeiture of the game.

14. **One Minute Time Limit:** Each player must complete a turn within one minute. If a player fails to complete a turn within one minute, a Mathematics Pentathlon® official will restore the gameboard to where all fraction bars were positioned at the beginning of the turn and the player forfeits that turn. The third violation of this rule results in

forfeiture of the game. Players may not challenge each other over this rule at the Mathematics Pentathlon.® An official at the tournament will decide whether or not this rule is being violated.

15. **Game Series Time Limit:** At the Mathematics Pentathlon, if the sixty (60) minutes of game series time for this event has run out before either player has established a win, the official at the tournament will determine a win or tie on the following basis. If time is called before a Mathlete™ has completed a turn, that turn will be allowed to be completed. The tricks accumulated to that point in the game will be determined. The Mathlete holding the most tricks will be declared the winner of that game. A win or tie for the event will then be determined by rules 11 and 12. If a series of only two games are completed in the allotted time with one player winning one game and the opposing player winning the second, a tie is declared. If a series of only two games is played with one player winning one game and a tie occurring in the second game, a win will be granted to the player who won one of the games.
16. **Calculators:** Calculators may be used at the Mathematics Pentathlon in Division IV, provided each Mathlete has one. If one player does not have a calculator, the opposing player may not use a calculator.
17. **Additional Fraction Bars:** Should a player "accidentally" overturn or draw more fraction bars than allowed, the additional fraction bars must be added to the player's discard pile.

GAMEPIECES FOR FRAC FACT

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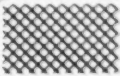
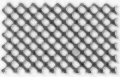
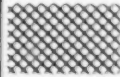
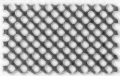
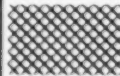
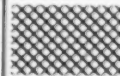
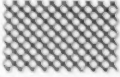
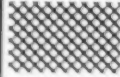
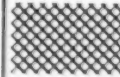
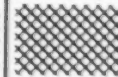
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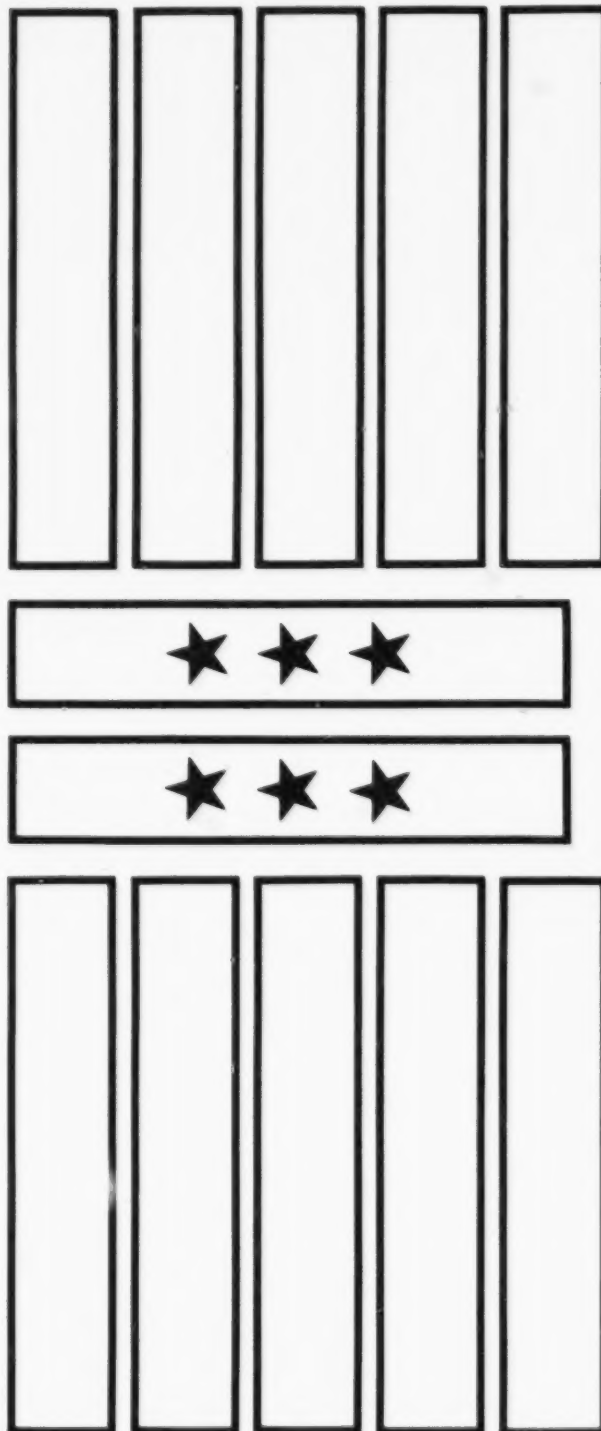
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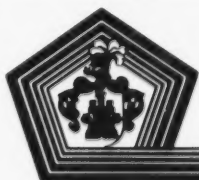
**FRAC
FACT**



DRAW PILE

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FRACTION PINBALL™

Created by
Dr. John C. del Regato

The ricocheting action of pinball is simulated in the Mathematics Pentathlon® event involving the addition and subtraction of fractions. Mathletes™ are challenged to become pinball wizards by converting fractions to decimal form and being the first to achieve the winning score. The use of blocking chips gives each player an opportunity to dramatically alter the game.

UNDERLYING

MATHEMATICS/SCIENCE: Computational reasoning involving fractions, fraction-decimal conversion, addition and subtraction of decimals, greater-than, less-than, inequality, spatial reasoning involving directionality and sequencing of positions; observation skills, experimentation, deductive and inductive reasoning, active problem solving

MATERIALS:

Official Mathematics Pentathlon gameboard, two (2) pawn "pinballs" and three (3) "blocking" chips of one color, two (2) pawn "pinballs" and three (3) "blocking" chips of a different color, and pencil and paper

TIME:

Total tournament playing time is one (1) hour. Each player has one minute to complete a turn. Prior to tournament competition, when first learning the game, more time should be granted.

GOAL:

To win, a player must be the first to bring the accumulated total to a score between 4.95 and 5.05. The score can not equal 4.95 or 5.05, but must be greater than 4.95 and less than 5.05.

TO START:

- A. Designate one of the two colors as the beginning player's color. Place one pawn of each color into a container and have one player select, without looking, one of the pawns. If the pawn selected is the color designated for beginning the game, the player who selected it begins. If not, the opposing player goes first.
- B. The blocking chips are stacked one on top of another alternating in color. This stack of chips is placed on the center pentagon from which ten paths radiate. The chip with the starting color should be on top of this stack and the chip on the bottom of this stack should be the opponent's color.
- C. The beginning player places a pawn pinball onto the starting position designated on the gameboard.

RULES:

PHASE I PINBALL PLACEMENT

1. **Alternating Turns and Pinball Placement:** Players alternate taking turns by placing a pinball into the starting position and then moving it along a straight-lined path.
2. **Nodes:** Nodes are the small dark circular junctions located at eleven intersections on the gameboard. When a player's pinball arrives at a node it may be redirected along one of the alternative straight lined paths. Within any given turn, the pinball may not be redirected through the path from which it arrived at a node.
3. **Accumulated Score:** Scores are derived as described in rule 4. Both players work off the same total and add their score, in decimal form, to the previous total.
4. **Fraction Circles:** Once a player's pinball has entered a circular region containing a fraction, the pinball must remain in that fraction circle until Phase III. The player must announce the decimal equivalent of the fraction represented within the circle. The fraction is added, in decimal form, to any prior accumulated total and the new total is announced by the player. Negative values reduce the total and positive values increase the total.
5. **Single Occupancy:** Fraction circles may be occupied by only one pinball at any time.
6. **Ricochet:** A pinball changes direction if it strikes the edge of the gameboard, but it must continue along the designated path.
7. **No Nodes No Directional Turn:** Players may not change the direction of their pinballs at places on the gameboard where paths cross but no nodes exist. The pinball must continue moving along the designated straight line path or ricochet off the edge of the gameboard until it arrives at a fraction circle.
8. **No Blocking Chip Movement:** Blocking chips are not to be moved during Phase I (see rule 12).
9. **End of Phase I:** Phase I has ended when both players have placed both of their pinballs into fraction circles and an accumulated total has been announced.

PHASE II BLOCKING CHIP PLACEMENT

10. **Placement of Blocking Chips:** After all four pinballs have been positioned, the beginning player removes the top blocking chip from the center stack and slides it along a path until it reaches a node. The blocking chip may be slid through any unoccupied fraction circle and may be redirected at a fraction circle along a chosen path.

A blocking chip may not be redirected through the path from which it arrived at a fraction base within the same turn. A player's blocking chip must stop and be placed onto the first unoccupied node encountered. Players alternate turns placing blocking chips until all six blocking chips are placed onto unoccupied nodes. Blocking chips must reach their final destination by traversing paths directly connected to a node or passing through unoccupied fraction circles and selecting unobstructed paths to reach an unoccupied node. A blocking chip may also change direction if it strikes the edge of the game-board, but it must continue along the designated path if it does so.



11. **Occupied Nodes and Fraction Circles:** No blocking chip may be moved into or through an occupied node or fraction circle (see rule 16 for exception to this rule in Phase III).
12. **No Pinball Movement in Phase II:** Only blocking chips are to be moved off the center stack during Phase II. During the tournament, a Mathematics Pentathlon® official will not allow a player to move a pinball during this phase.

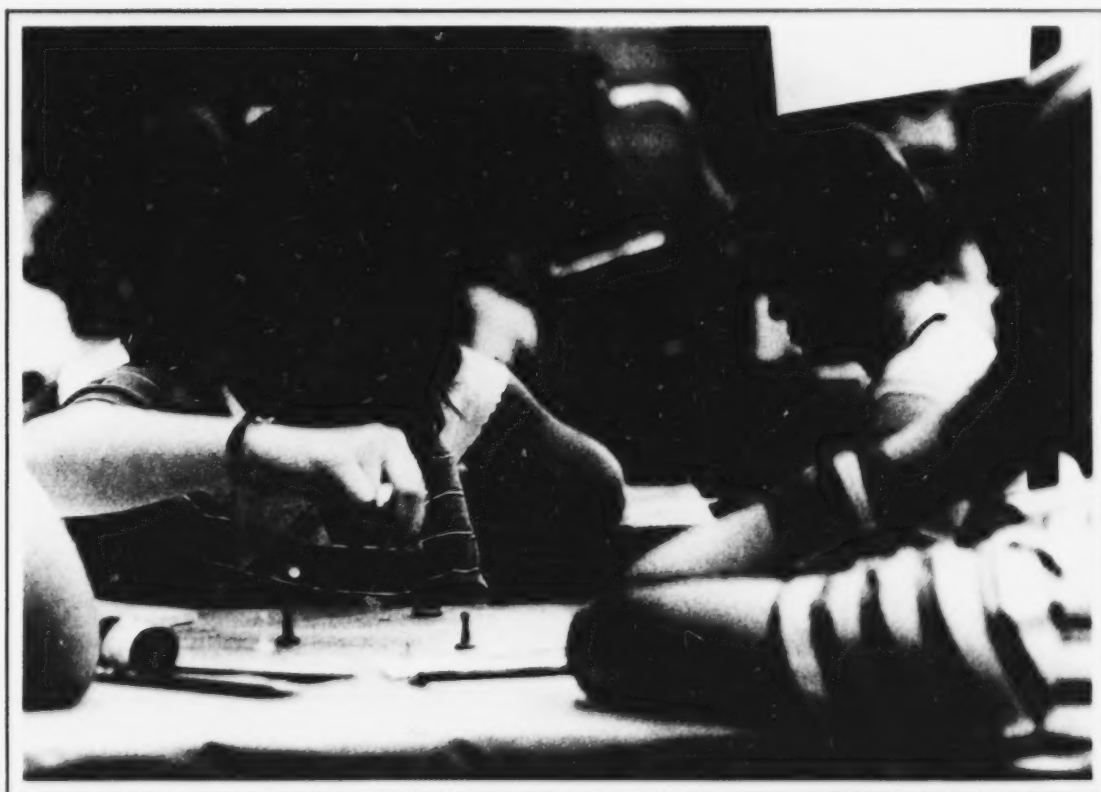
Also, blocking chips already removed from the center of the gameboard may not be moved a second time until Phase III begins. A player's third violation of Phase II and Phase I movement and placement rules results in forfeiture of the game.

13. **No Neutralization:** Players may not exercise the neutralization rule (see rule 16) during Phase II.
14. **End of Phase II:** Phase II ends when the second player has removed the last blocking chip from the center stack and positioned it on a node.

PHASE III PINBALL OR BLOCKING CHIP MOVEMENT

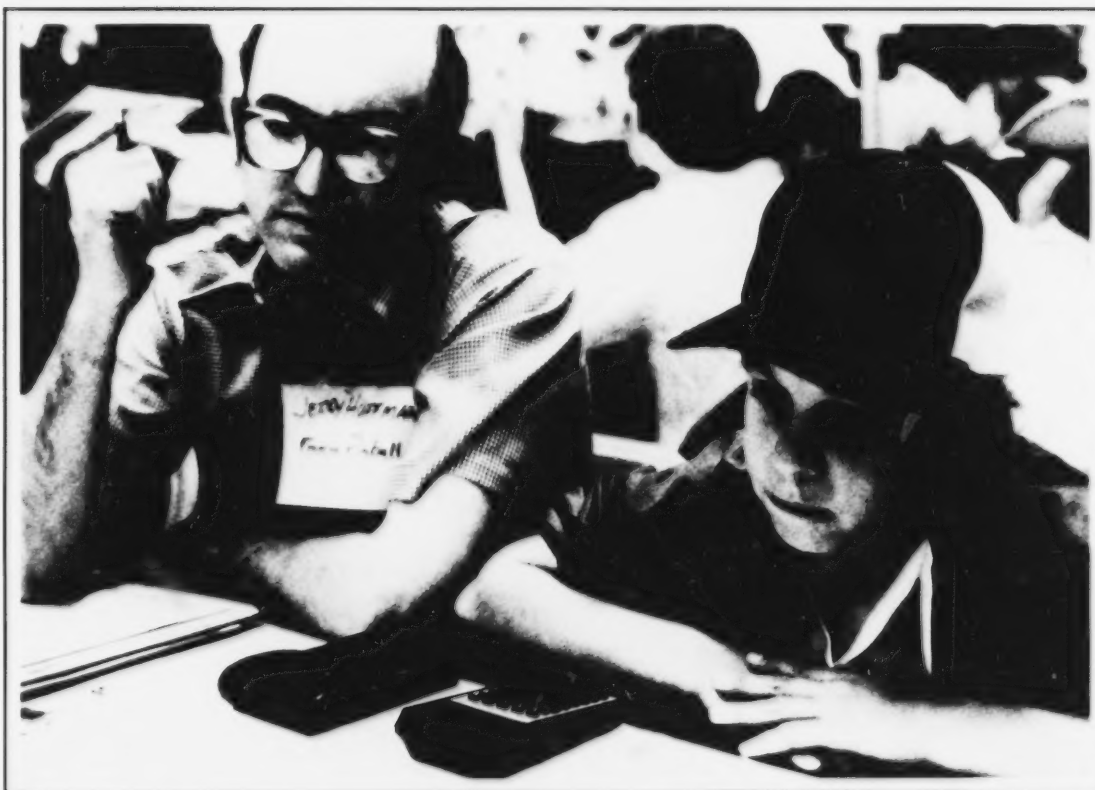
15. **Moving Pinballs or Blocking Chips:** After the second player has ended Phase II, the beginning player can either move a pinball or a blocking chip within a turn. Players alternate turns by moving their pinballs or their blocking chips according to the descriptions of movement and placement above. One of a player's pinballs can be moved into a new position with the announced decimal equivalent to the fraction added to the accumulated total. The new total is also announced. A player may choose instead to move a blocking chip onto a different





node or to neutralize (see rule 16) an opposing player's blocking chip. Pinballs may not be moved through nodes occupied by a blocking chip. Pinballs may not move into occupied fraction circles.

16. **Neutralization:** During Phase III a player may move a blocking chip onto a node occupied by an opponent's blocking chip. This node in effect becomes neutralized and the two blocking chips are stacked onto the start circle. The chip belonging to the player making the neutralization move appears on top of the stack. This neutralization move can be executed even if other blocking chips are already stacked on the *start* circle. The additional chips are positioned as described above and placed on top of the existing stack.
17. **Beginning a Turn:** A player's turn begins when an opponent has moved a pinball or blocking chip into a new location and is no longer touching the playing piece. In the case of neutralization, the turn begins when an opponent is no longer touching the blocking chips placed onto the start circle.



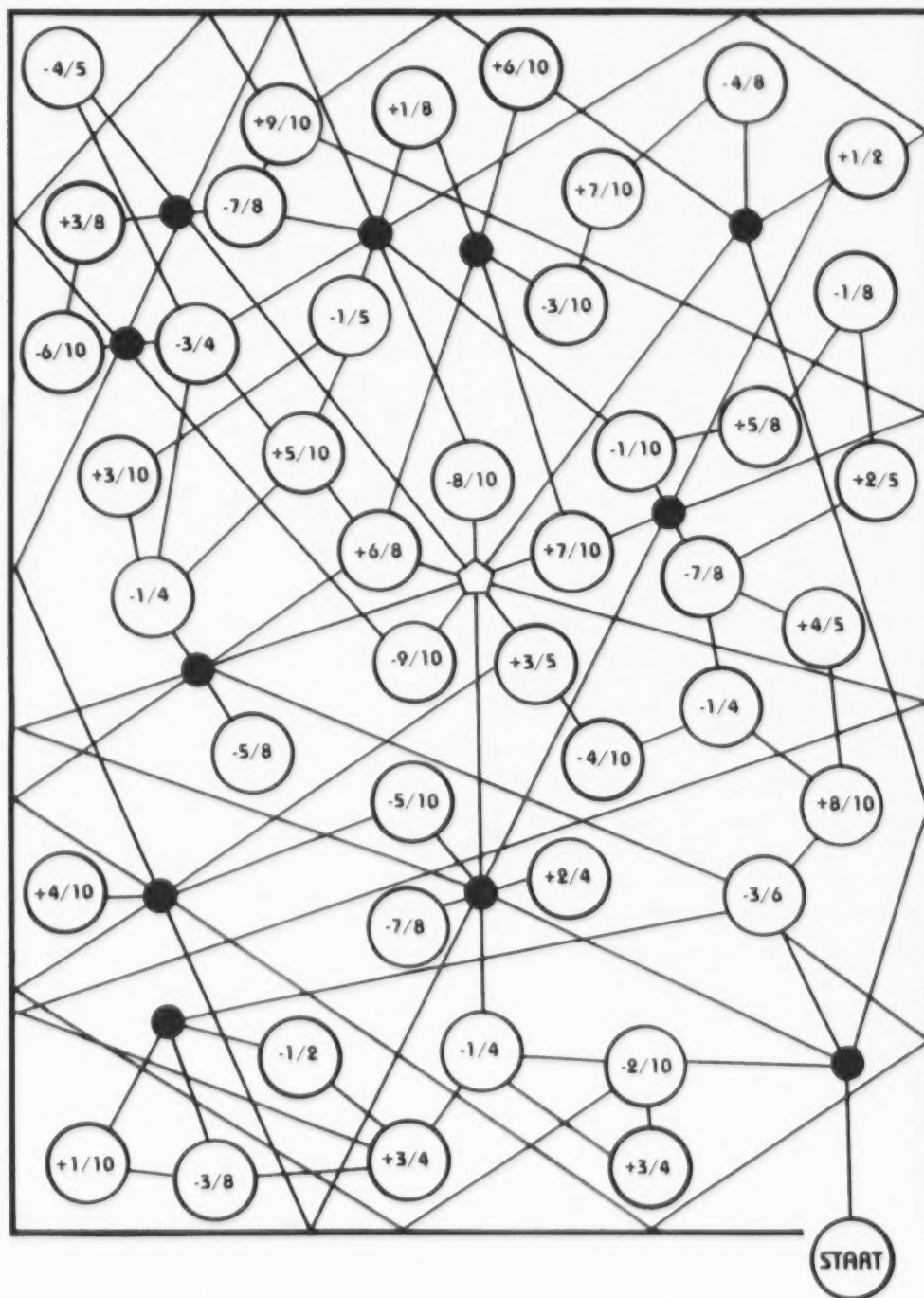
18. **End of Turn:** A player's turn ends when the player's hand is removed from a pinball or blocking chip which has been moved to a different location. In the case of neutralization, the turn ends when a player is no longer touching the blocking chips placed onto the start circle.
19. **Touch-Move:** Once a pinball or blocking chip has been touched, it must be moved by the player during that turn. Failure to move a touched piece results in forfeiture of the game.
20. **Win:** To win, a player must succeed at bringing the accumulated score to a value between 4.95 and 5.05 ($4.95 < x < 5.05$) and announcing that this goal has been attained.
21. **Tie:** A tie is declared if during Phase III the accumulated score dips to 1 or below or rises to 9 or above.
22. **Challenges:** If a player violates any of the preceding rules, the opposing player may issue a challenge after a player has ended a turn. To indicate that a challenge is being made, a player must announce "challenge". A Mathematics Pentathlon® official must rule on this challenge. In order to do so, an official time out will be granted

TOURNAMENT RULES:

and the official will ask the challenger to state the nature of the challenge. The official must rule on whether the challenge, as stated by the player, is correct or incorrect. If the challenge is found to be correct, the challenger may place one of the opponent's blocking chips onto the starting circle. The gameboard will be restored to where the pinballs and blocking chips were positioned at the beginning of the challenged player's turn. The challenged player's score will be returned to its previous value and the challenger will begin a turn. If the challenge is found to be incorrect, the challenger will forfeit a turn and the challenged player may place one of the challenger's blocking chips onto the starting circle. Three consecutive incorrect challenges result in forfeiture of the game.

23. **One Minute Time Limit:** Each player must complete a turn within one minute. If a player fails to complete a turn within one minute, a Mathematics Pentathlon® official will restore the gameboard to where all pinballs and blocking chips were positioned at the beginning of that turn and the player forfeits that turn. The third violation of this rule results in forfeiture of the game. Players may not challenge each other over this rule at the Mathematics Pentathlon. An official at the tournament will decide whether or not this rule is being violated.
24. **Game Time Limit:** At the Mathematics Pentathlon, if the sixty (60) minutes of game time have run out before either player has won the game, a tie is declared. If a player has touched a pinball or blocking chip and is in the process of moving it when game time is called, the player will be allowed to complete that turn.
25. **Calculators:** Calculators may be used at the Mathematics Pentathlon in Division IV, provided each Mathlete™ has one. If one player does not have a calculator, the opposing player may not use a calculator.

FRACTION PINBALL

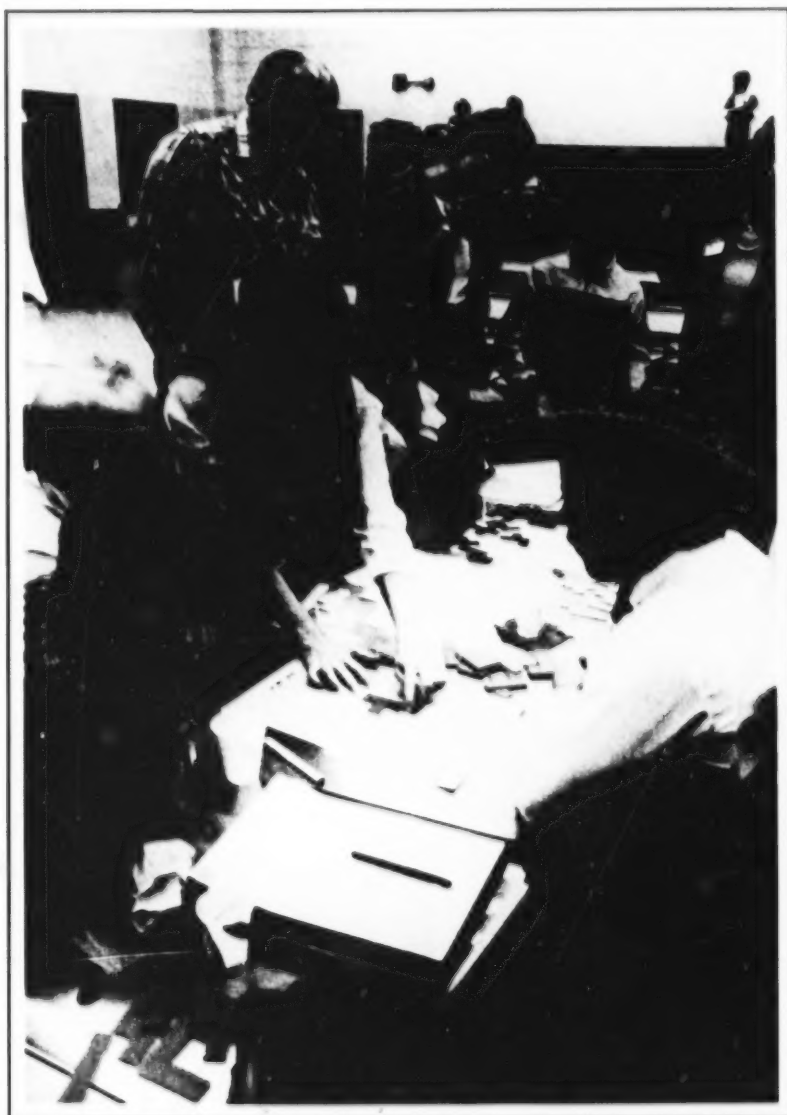


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*Based on a point system, the Mathematics Pentathlon® celebrates the accomplishments of **many** mathletes. Regardless of the outcome, all participants are considered "winners". One only "fails" when one does not attempt to grow. In this attempt, an individual may discover that an opponent contributed other insights that one did not consider. Indeed, **real learning** has just been experienced!*







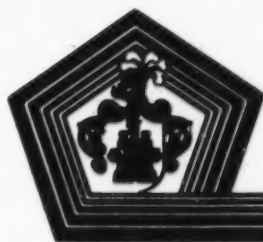
Paula Minton
One of our star volunteers.

The Pentathlon Institute offers Mathematics Pentathlon® orientation sessions or special in-service sessions on teaching Mathematics Experienced-Based Approach (MEBA™). For further information please contact:

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National Director, Mathematics Pentathlon®
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